Roll No. ....

E–767

M. A./M. Sc. (Third Semester)

EXAMINATION, Dec.-Jan., 2020-21

MATHEMATICS

(Optional Paper)

Paper Fourth (A)

(Operations Research—I)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. The name Operations Research is first coined in the year :

   (a) 1945

   (b) 1935

   (c) 1940

   (d) 1950

P. T. O.

2. The objectives of Operations Research is :
   - (a) To find new methods of solving problems
   - (b) To derive formulas of solving problems
   - (c) To utilize the services of scientists
   - (d) Optimum utilization of existing resources

3. In L. P. P., if there is no non-negative replacement ratio in a solution which is sought to be improved, then the solution is :
   - (a) Feasible
   - (b) Infeasible
   - (c) Degenerate
   - (d) Unbounded

4. In a maximization L. P. P., if at least one artificial variable is in the basis, but not at zero level and the coefficient of M in each of the net evaluation is non-negative, then we have :
   - (a) Optimum solution
   - (b) Feasible solution
   - (c) No feasible solution
   - (d) Unbounded solution

5. If the primal has an unbounded solution, then the dual has :
   - (a) Optimal solution

(b) No feasible solution

(c) Bounded solution

(d) None of the above

6. If Dual has a finite optimum solution, then the primal has :

(a) Not have a finite optimum solution

(b) Have only basic feasible solution

(c) Have a finite optimum solution

(d) None of the above

7. Identify the statement which is not correct :

(a) Post-optimality analysis is normally carried out after the optimum solution is reached.

(b) Adding of a new variable may disturb the feasibility of the current optimum solution.

(c) Adding of a constraint may affect the current optimum solution.

(d) Deletion of an existing variable may affect the feasibility of the current optimum solution.

8. Identify the statement which is not correct :

(a) Post-optimality analysis forms an integral part of formulating a linear programming problem.

(b) The feasibility of the current optimum solution may be affected if right hand side of the constraint is changed.

(c) The optimality of the current optimum solution may be affected if right hand side of the constraint is changed.

(d) The optimality of the current optimum solution may be affected if the coefficient of the objective function is changed.

9. In linear programming context, parametric programming is a technique to :

(a) Investigate the effect of simultaneous changes of all components of right hand side of constraints and coefficients of variables in the objective.

(b) Minimize cost of operations.

(c) Determine an optimum solution of an L. P. P.

(d) None of the above

10. In case of unbounded variables in bounded variable algorithm, lower and upper bound of decision variable are :

(a) ∞ and ∞ respectively

(b) ∞ and 0 respectively

(c) 0 and ∞ respectively

(d) None of the above

11. In bounded variable algorithm, if a non-basic variable enters the basis, its value should :

    (a) Exceed its upper bound

    (b) Not exceed its upper bound

    (c) Exceed its lower bound

    (d) None of the above

12. Identify the statement which is not correct in the context of bounded variable algorithm :

    (a) Lower bound of a decision variable can always be converted into non-negative decision variable.

    (b) Upper bound of a decision variable can always be converted into non-negative decision variable.

    (c) In a L. P. P., if any of the variable is at a positive lower bound, substitute it out at its lower bound.

    (d) None of the above

13. The total number of allocation in a basic feasible solution of transportation problem of $m \times n$ size is equal to :

    (a) $m \times n$

    (b) $(m/n) - 1$

    (c) $m + n - 1$

    (d) $m + n + 1$

14. To solve degeneracy in the transportation problem we have to :

    (a) Put allocation in one of the empty cell as zero.

    (b) Put a small element epsilon in any one of the empty cell.

    (c) Allocate the smallest element epsilon in such a cell, which will not form a closed loop with other loaded cells.

    (d) Allocate the smallest element epsilon in such a cell, which will form a closed loop with other loaded cells.

15. Assignment Problem is basically a :

    (a) Maximization problem

    (b) Minimization problem

    (c) Transportation problem

    (d) None of the above

16. The assignment problem will have alternate solutions when total opportunity cost matrix has :

    (a) At least one zero in each row and column.

    (b) All rows have two zeros.

    (c) There is a tie between zero opportunity cost cells.

    (d) Two diagonal elements are zeros.

17. Choose the relationship which is not true :

    (a) Free float + Independent float = Total float

    (b) Free float + Head event slack = Total float

    (c) Free float + Interfering float = Total float

    (d) None of the above

18. PERT analysis computes the variance of the total project completion time as :

    (a) The sum of the variances of all activities in the project

    (b) The sum of the variances of all activities not the critical path

    (c) The variance of the final activity of the project

    (d) The sum of the variances of all activities on the critical path

19. A dummy activity is required when :

    (a) Two or more activities have the same starting events

    (b) Two or more activities have the different starting events

    (c) Two or more activities have the same ending events

    (d) The network contains two or more activities that have identical starting and ending events

20. Which of the following is not a rule for network construction ?

    (a) A network should have only initial and one terminal node.

(b) Identical initial and final nodes can identify two activities.

(c) All activities must be tied into a network. Dangers must be avoided.

(d) A network should employ only those dummy activities which are absolutely necessary.

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Define Operations Research.
2. Define Linear programming problem.
3. Define duality in L. P. P.
4. Define sensitivity analysis in L. P. P.
5. Define goal programming.
6. Define assignment problem.
7. Define transportation problem.
8. What is the basic object of CPM technique ?

Section—C 3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. What are the situations, where Operations Research techniques will be applicable ?
2. Briefly explain the characteristics of Operations Research.
3. Prove that dual to the dual is the primal.

4. Formulate the duel to the following linear programming problem :

Maximize :

$$z = 5x_1 + 3x_2$$

Subject to the constraints :

$$3x_1 + 5x_2 \leq 15$$

$$5x_1 + 2x_2 \leq 10$$

$$x_1, x_2 \geq 0 .$$

5. Solve the following linear goal programming problem graphically :

Find x_1 and x_2 so as to :

Minimize :

$$z = G_1 \quad d_3^+ + d_4^+ \quad + G_2 d_1^+$$

$$+ G_3 d_2^- + G_4 \left(d_3^- + \frac{3}{2} d_4^- \right)$$

and satisfy the goals :

$$G_1 : x_1 + x_2 + d_1^- - d_1^+ = 40$$

$$G_2 : x_1 + x_2 + d_2^- - d_2^+ = 100$$

$$G_3 : x_1 + d_3^- - d_3^+ = 30$$

$$G_4 : x_2 + d_4^- - d_4^+ = 15$$

$x_i, d_i^-, d_i^+ \geq 0$ for all $i = 1, 2, 3, 4.$

The goal have been listed in order of priority.

6. Explain Interior point algorithm.
7. Explain north-west corner rule for transportation problem.
8. What is the differentiate between CPM network and PERT network ?

Section—D 4 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. Use penalty method of solve the following L. P. P. :

Maximize :

$$z = x_1 + x_2 + x_4$$

Subject to the constraints :

$$x_1 + x_2 + x_3 + x_4 = 4$$

$$x_1 + 2x_2 + x_3 + x_5 = 4$$

$$x_1 + 2x_2 + x_3 = 4$$

$$x_j \geq 0 \text{ for } j = 1, 2, 3, 4, 5$$

Or

Use two-phase simplex method of solve the following L. P. P. :

Maximize :

$$z = 3x_1 + 2x_2$$

Subject to the constraints :

$$2x_1 + x_2 \leq 2$$

$$3x_1 + 4x_2 \geq 12$$

$$x_1, x_2 \geq 0 .$$

2. Using dual simplex method to solve the following linear programming problem :

Minimize :

$$z = x_1 + 2x_2 + 3x_3$$

Subject to the constraints :

$$x_1 - x_2 + x_3 \geq 4$$

$$x_1 + x_2 + 2x_3 \leq 8$$

$$x_2 - x_3 \geq 2$$

$$x_1, x_2, x_3 \geq 0 .$$

Or

Consider the linear programming problem :

Minimize :

$$z = 3x_1 + 6x_2 + x_3$$

Subject to the constraints :

$$x_1 + x_2 + x_3 \geq 6$$

$$x_1 + 5x_2 - x_3 \geq 4$$

$$x_1 + 5x_2 + x_3 \geq 24$$

$$x_1, x_2, x_3 \geq 0 .$$

Then

(a) Solve the L. P. P.

(b) Discuss the effect of changing the requirement vector from [6, 4, 24] to [6, 2, 12] on the optimum solution.

3. Using the bounded variable technique, solve the following linear programming problem :

Maximize :

$$z = x_2 + 3x_3$$

Subject to the constraints :

$$x_1 + x_2 + x_3 \leq 10$$

$$x_1 - 2x_3 \geq 0$$

$$2x_2 - x_3 \leq 10$$

$$0 \leq x_1 \leq 8$$

$$0 \leq x_2 \leq 4$$

$$x_3 \geq 0\,.$$

Or

Consider the parametric linear programming problem :

Maximize :

$$z = \lambda - 1 \;\; x_1 + x_2$$

Subject to the constraints :

$$x_1 + 2x_2 \le 10$$

$$2x_1 + x_2 \le 11$$

$$x_1 - 2x_2 \le 3$$

$$x_1, x_2 \ge 0\,.$$

Perform complete parametric programming analysis. Identify all critical values of the parameter λ and all optimum basic solutions.

4. Obtain an optimum basic feasible solution to the following transportation problem :

To

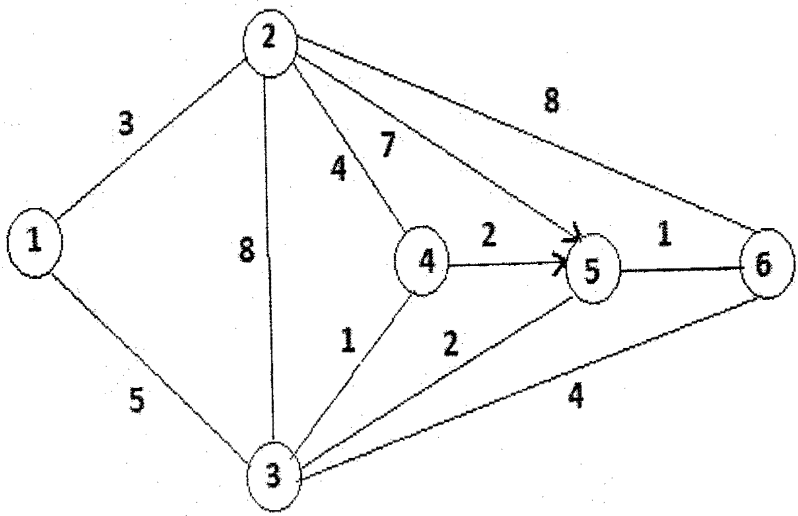
From	7	3	4	2	Available
	2	1	3	3	
	3	4	6	5	
Demand	4	1	5		

Or

Solve the following assignment problem :

	I	II	III	IV	V
1	3	8	2	10	3
2	8	7	2	9	7
3	6	4	2	7	5
4	8	4	2	3	5
5	9	10	6	9	10

5. Consider the distance network as shown below :



Then :

(a) Apply Floyd's algorithm to it an generate the final distance matrix and prcedence matrix.

(b) Find the shortest path and the corresponding distance from the source node to the destination node as indicated in each of the cases :

1 – 6, 5 – 1 and 5 – 2

Or

The following table lists the jobs of a network along with their time estimates :

Job	Duration (days)			
i	*j*	Optimistic	Most likely	Pessimistic
1	2	3	6	15
1	6	2	5	14
2	3	6	12	30
2	4	2	5	8
3	5	5	11	17
4	5	3	6	15
6	7	3	9	27
5	8	1	4	7
7	8	4	19	28

Then :

(a) Draw the project network diagram.

(b) Calculate the length and variance of the critical path after estimating the earliest and latest event time for all nodes.